

# What Does it Take to Disconnect a P2P Network?

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# Outline

- 1 Background
  - P2P Basics
  - Motivation
- 2 Global P2P Resilience
  - Classical Results
  - Lifetime-Based Extension
- 3 Lifetime-Based Resilience
  - Assumptions
  - Expected Time to Isolation
  - Probability of Isolation
  - Effect of Varying Node Degree
- 4 Conclusion

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# P2P Basics

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- *Local*: node isolation

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## Resilience Metrics

- *Local*: node isolation
- *Global*: disconnection of the graph (its undirected version)

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# Motivation I

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## P2P Churn

- Static failure model is rarely applicable to real P2P networks (Gnutella, KaZaA, etc.)
- Nodes depart asynchronously based on user browsing habits or interest (*dynamic* resilience)

# Motivation II

## Common Static Resilience Model

Most resilience results are local:  $P(\text{node } v \text{ is isolated}) = p^k$ ,  
where  $k$  is the degree of  $v$

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- Can we do better?

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# Classical Results I

## Disconnection of Random Graphs



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- *Almost every* (i.e., with probability  $1 - o(1)$  as  $n \rightarrow \infty$ ) random graph  $G(n, p)$ ,  $G(n, M)$ ,  $G(n, k_{out})$  remains connected after static failure if and only if it has no isolated vertices

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- *If each user manages to avoid isolation, the graph almost surely remains connected after the failure!*

## Intuition

Conditional probability of partitioning along a set boundary *while* not developing isolated nodes tends to zero

# Classical Results II

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- This can be extended to any graph with similar or better node expansion properties (Chord, CAN, Pastry, etc.)

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Table: Chord with  $n = 16384$  under  $p$ -percent failure

$p$	$P(G \text{ is connected})$	$P(\text{no isolated nodes})$
0.5	0.99996	0.99996
0.6	0.99354	0.99354
0.7	0.72619	0.72650
0.8	0.00040	0.00043



## Classical Result III

### Application to P2P graphs

All tested P2P graphs (Chord, Symphony, CAN, Pastry, Randomized Chord, de Bruijn, and several unstructured random graphs) remained connected almost surely as long as they did not have an isolated node

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### Milestone

Local resilience of P2P networks *under static failure* implies their global resilience

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# Lifetime-Based Extension I

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Compute  $P(G \text{ survives } N \text{ user joins without disconnecting})$

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Let  $Y$  be the number of user joins before the first disconnection of the network

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## Simple Model

- For almost every sufficiently large graph:

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- *Knowledge of  $\phi$  is all we need to understand dynamic resilience of P2P systems!*

# Lifetime-Based Extension III

## Example

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Search time (min)	Actual $P(Y > N)$	Model
6	0.9732	0.9728
7.5	0.8118	0.8124
8.5	0.5669	0.5659
9	0.4065	0.4028
9.5	0.2613	0.2645
10.5	0.0482	0.0471

Table: Comparison of model to simulations for  $N = 10^6$  user joins

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# Overview of Lifetime Model I

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- *Neighbor replacement*: once a failed neighbor is detected, a replacement search is performed

# Overview of Lifetime Model II

## Definition

A node becomes *isolated* when all of the neighbors in its table are simultaneously in the failed state

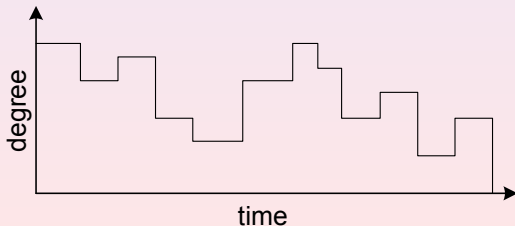
# Overview of Lifetime Model II

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## Degree Evolution

User degree  $W(t)$  is a random process shown below





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Let  $R_i$  be the remaining (i.e., *residual*) lifetime of neighbor  $i$  when  $v$  joined the system

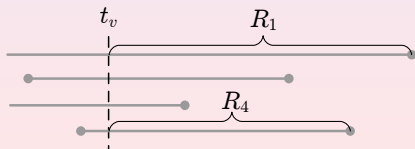
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## Definition

Let  $S_i$  be a random variable describing the total search time for the  $i$ -th replacement in the system

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- How does varying node degree between users improve/degrade resilience?
- How to increase resilience of existing systems?

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# Expected Time to Isolation I

## Definition

Let  $T$  be the random time instance when  $v$  becomes isolated (i.e.,  $T = \inf\{t > 0 : W(t) = 0\}$  is the first hitting time of  $W(t)$  on level 0)



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## Theorem

*Assuming relatively small search delays, the following approximation holds for all lifetime and search distributions:*

$$E[T] \approx \frac{E[S_i]}{k} \left[ \left( 1 + \frac{E[R_i]}{E[S_i]} \right)^k - 1 \right]$$

# Expected Time to Isolation II

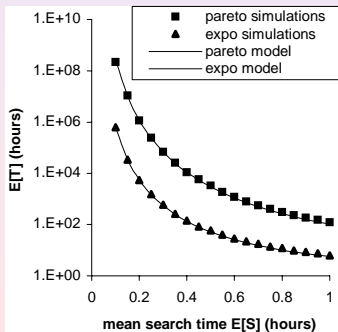
## Simulations

Simulations with average lifetime 30 minutes and  $k = 10$  for a 1000 node system (four distributions of  $S_i$ )

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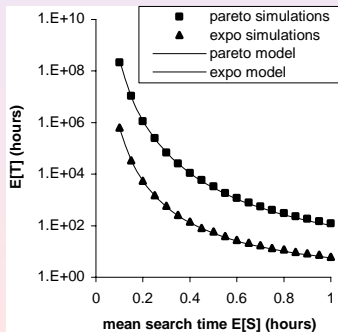
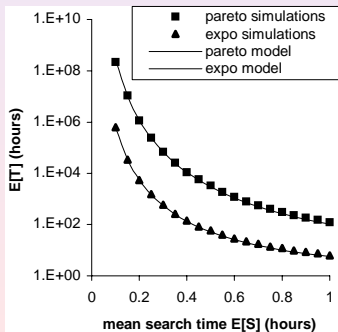
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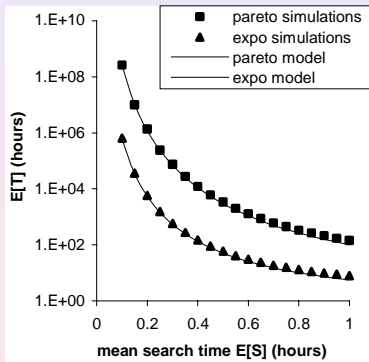
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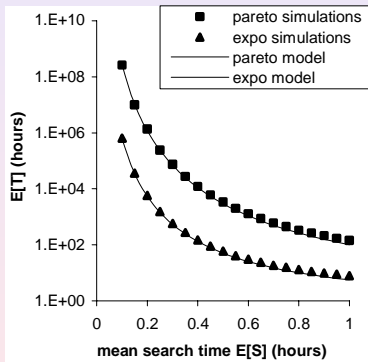


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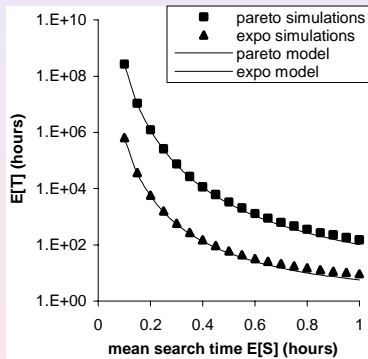


(g) exponential  $S_i$

# Expected Time to Isolation III



(i) exponential  $S_i$



(j) Pareto  $S_i$  with  $\alpha = 3$

# Expected Time to Isolation IV

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## Result for Chord

We immediately obtain from the main model:

$$E[T] \approx \frac{\delta + d \log_2 n}{2k} \left( 1 + \frac{2E[R_i]}{\delta + d \log_2 n} \right)^k$$



# Expected Time to Isolation V

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Chord with  $n = 1$  million,  $d = 200$  ms,  $E[R_i] = 1$  hour  
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# Expected Time to Isolation V

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(Pareto lifetimes with  $E[L_i] = 30$  minutes)

Timeout $\delta$	$k = 20$	$k = 10$	$k = 5$
20 sec	$10^{41}$ years	$10^{17}$ years	188,034 years
2 min	$10^{28}$ years	$10^{11}$ years	282 years
45 min	404,779 years	680 days	49 hours

Table: Expected time  $E[T]$  to isolation

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## General Idea

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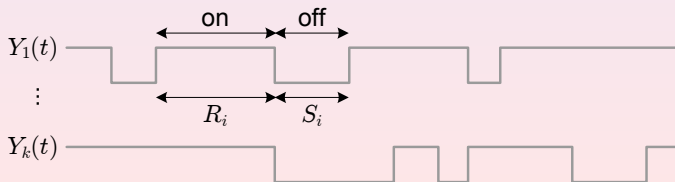
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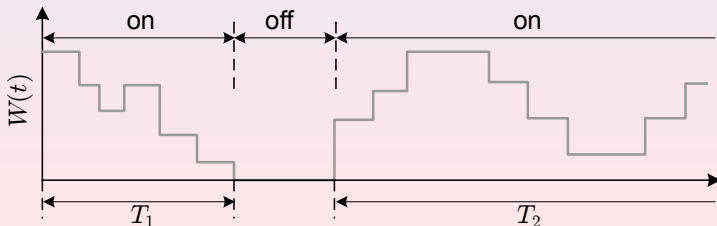
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# Probability of Isolation III

## Theorem

For exponential lifetimes and  $E[S_i] \ll E[L_i]$ , the probability of isolation  $\phi$  can be approximated by:

$$\phi \approx \frac{E[L_i]}{E[T]} = \frac{\rho k}{(1 + \rho)^k + \rho k - 1}$$

where  $\rho = E[L_i]/E[S_i]$

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## Verification

Simulations match the model very well and, for small  $S_i$ , the results are not sensitive to the distribution of search delay

# Probability of Isolation IV

## Simulations

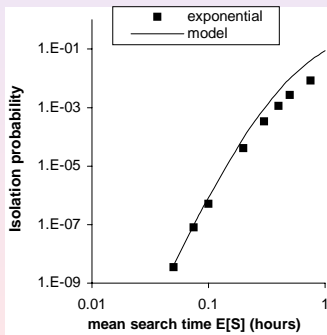
System with  $E[L_i] = 0.5$  hours and  $k = 10$



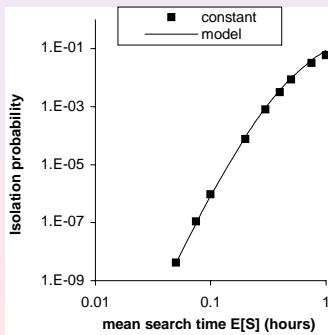
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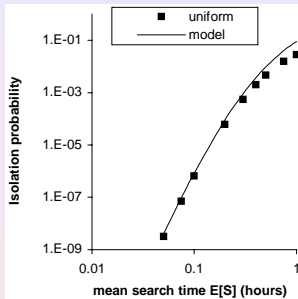


(m) exponential  $S_i$

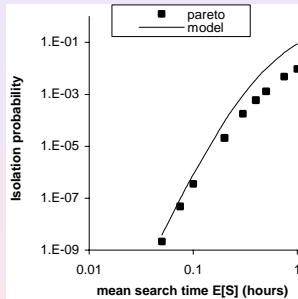


(n) constant  $S_i$

# Probability of Isolation V

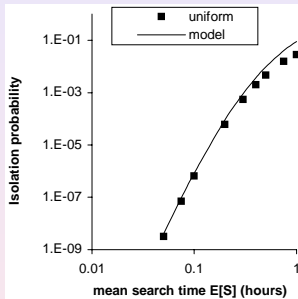


(o) uniform  $S_i$

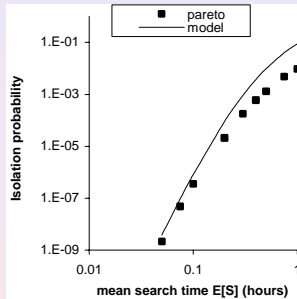


(p) Pareto  $S_i$  with  $\alpha = 3$

# Probability of Isolation V



(q) uniform  $S_i$



(r) Pareto  $S_i$  with  $\alpha = 3$

## Observation

Heavy-tailed search delays are better than light-tailed

# Probability of Isolation VI

## Application to Pareto Lifetimes

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# Probability of Isolation VI

## Application to Pareto Lifetimes

- Notice that heavy-tailed (e.g., Pareto) lifetimes  $L_i$  imply stochastically larger residual lifetimes  $R_i$
- For example, shape parameter  $\alpha = 3$  leads to  $E[R_i] = 2E[L_i]$
- The exponential result can be used as an upper bound on  $\phi$  for heavy-tailed distributions of lifetime:

$$\phi \leq \frac{\rho k}{(1 + \rho)^k + \rho k - 1}$$

where  $\rho = E[L_i]/E[S_i]$

# Probability of Isolation VII

## Simulations

Table shows the minimum degree needed to guarantee a certain  $\phi$  under Pareto lifetimes with  $\alpha = 2.06$  and  $k = 10$

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$\phi$	Static $p = 1/2$	Lifetime node failure	Mean search time $E[S_i]$		
			6 min	2 min	20 sec
$10^{-6}$	20	Upper-bound model	10	7	5
		Simulations	9	6	4
$10^{-9}$	30	Upper-bound model	14	9	6
		Simulations	13	8	6



# Examples I

## Local Resilience Example

Consider a Chord system with  $n = 10^{11}$  nodes,  $E[L_i] = 30$  minutes, and  $E[S_i] = 1$  minute

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Classical analysis with  $p = 0.5$  requires  $k = 37$  to ensure that a given node remains connected w.p.  $1 - 1/n$

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## Lifetime Result

The same bound can be achieved with  $k = 9$  as long as the tail of the lifetime distribution is exponential or heavier

## Examples II

### Global Resilience Example

Consider CAN with exponential lifetimes (mean 30 minutes), degree  $k = 12$ , and  $n = 4096$  nodes

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Search time (min)	Actual $P(Y > N)$	Empirical $\phi$	Model $\phi$
6	.9732	.9728	.9728
7.5	.8218	.8224	.8215
8.5	.5669	.5659	.5666
9	.4065	.4028	.4016
9.5	.2613	.2645	.2419
10.5	.0482	.0471	.0424

Table: Comparison of model to simulations for  $N = 10^6$  user joins

## Examples III

### Global Resilience Example (continued)

Assume now that the mean search delay is 1-minute and that  $10^6$  users join/leave per day

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- Graph stays connected for 2,700 years w.p. 0.9956



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### Model Result

- Graph stays connected for 2,700 years w.p. 0.9956
- Mean delay between disconnections is 5.9 million years!

# Outline

- 1 Background
  - P2P Basics
  - Motivation
- 2 Global P2P Resilience
  - Classical Results
  - Lifetime-Based Extension
- 3 **Lifetime-Based Resilience**
  - Assumptions
  - Expected Time to Isolation
  - Probability of Isolation
  - **Effect of Varying Node Degree**
- 4 Conclusion

# Effect of Varying Node Degree I

## Degree Regularity and Resilience

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## Degree Regularity and Resilience

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- In particular, is Gnutella with heavy-tailed degree more resilient than DHTs?

## Theorem

*Under the assumptions made earlier, degree-regular graphs are the most resilient for a given average degree  $E[k_i]$*

## Varying Node Degree II

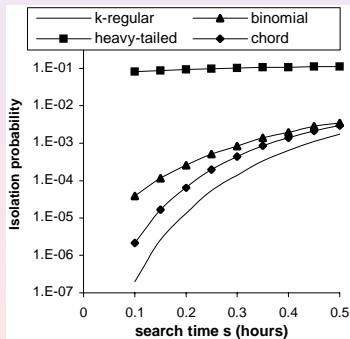
### Simulations

Examine three degree-irregular systems with average degree  $E[k_i] = 10$  and Pareto lifetimes with  $E[L_i] = 0.5$  hours

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Varying node degree from peer to peer can have a positive impact on resilience *only* when these decisions are correlated with lifetimes



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- Attach to neighbors with larger residual lifetime (age determines the expected residual lifetime of each user)
- Unstructured systems: sample  $2k$  users, sort by age, and choose top  $k$  to be your neighbors
- Structured: do not let users of age smaller than a certain threshold to be responsible for DHT space

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## Findings

- P2P systems under churn almost surely remain connected as long as no user suffers isolation from the system
- Under all practical search times,  $k$ -regular graphs are much more resilient than traditionally thought
- Increasing the expected residual lifetime  $E[R_i]$  of the neighbors is one simple way to improve resilience
- Future work: model in-degree, examine lifetime-dependent neighbor selection, take node capacity into consideration